水資源系統串接之傅立葉頻譜網流最佳化分析

Fourier Spectrum Network Optimization Analysis of Pipeline Connectivity for Water Resource Systems in Northern Taiwan

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摘 要

本研究聚焦於在氣候變遷影響對臺灣水資源管理面臨的挑戰。台灣雖然擁有良好的氣候條件，擁有豐沛的降雨量，但也因地理條件不良，使水資源難以保存，構成台灣獨特的水文情況，大量的水資源流入的同時，也有大量的水資源流失，只保留小部份的水資源在地區上。再加上社會發展的情況下，農業、民生、工業用水的水資源的競合已是重要的議題。而近年來，在氣候變遷影響下，台灣發生極端水文事件的頻率增加，使降雨的頻率更為集中，強度也更高，使台灣更難保留水資源，更為加劇農業與科技產業之間的水資源分配競爭。本研究的貢獻為建構創新的傅立葉頻譜網流最佳化模型，討論水資源系統幹管串連對供水能力與抗旱能力的影響，並分析如何透過跨區域水資源調配提升供水穩定性，並討論水資源系統串接的運作效率與供水能力的不確定性。此模型包含將建立之最小成本網路規劃模型，針對水資源供應系統的流量守恆、管線容量與成本效益進行優化分析，並整合傅立葉轉換解析不同水源串接的供水時間序列，以提供更精確的水資源調配策略。

關鍵詞：網流最佳化，傅立葉頻譜分析，水資源調配

Abstract

This study examines the challenges posed by climate change on water resource management in Taiwan, a region characterized by abundant rainfall yet constrained by complex geographical conditions that hinder effective water retention. Despite substantial inflows of water resources, a significant portion is lost due to rapid surface runoff, leaving only a limited amount available for local use. Additionally, as Taiwan undergoes socioeconomic development, competition for water resources among the agricultural, industrial, and residential sectors has intensified, creating significant management challenges. In recent years, climate change has led to an increased frequency of extreme hydrological events, resulting in more concentrated and intense rainfall, further exacerbating water retention difficulties and intensifying conflicts over water allocation.

The significant contribution of this study is to develop a new spectrum network optimization model. The model analyzes the impact of integrating water resource system trunk connections on water supply capacity and drought resilience, analyzing how cross-regional water resource allocation can enhance supply stability. This newly proposed model integrate a network flow model with Fourier spectrum analysis of hydrological time series data to assess system efficiency and the uncertainty of water supply capacity. In this model, the minimal cost network programming model is developed to optimize flow conservation, pipeline capacity, and cost-effectiveness within the water supply system. By integrating Fourier Transform, this study analyzes the time-series behavior of interconnected water sources, providing a more precise and effective water resource allocation strategy. The findings aim to contribute to improved water management policies and infrastructure planning, ensuring the sustainable utilization of Taiwan's water resources in the face of increasing climate variability and competing sectoral demands.

Keywords: network optimization, Fourier spectrum analysis, water resource management.

一、前言

Taiwan, located in the northwestern Pacific, experiences distinct seasonal climates due to its geographic position. The northern region, situated north of the Tropic of Cancer, is influenced by ocean currents such as the Kuroshio and China Coastal Currents and has a subtropical monsoon climate, with heavy summer rainfall brought by the East Asian monsoon and humid winters caused by the northeast monsoon. The Central Mountain Range further intensifies precipitation by blocking cold air in winter and lifting moist summer air. In addition, Taiwan’s position at the boundary of tropical and subtropical zones makes it highly susceptible to tropical cyclones, with an average of three to four events per year, each bringing intense rainfall. Despite abundant precipitation, Taiwan’s steep terrain, with mountains covering a large portion of the land and average slopes of around 30 degrees, limits water retention. Accelerated surface runoff and reduced infiltration result in approximately 60 percent of rainfall flowing directly into the sea, making droughts common even during rainy periods and posing continuous challenges for water management and agricultural development.

Amid growing demand and limited resources, climate change has intensified water-related risks. Rainfall has become more concentrated and intense, increasing drought frequency. In 2021, Taiwan faced its most severe drought in a century. In response, the Water Resources Agency launched inter-regional pipeline projects to transfer water from surplus to deficit areas, strengthening supply resilience.

This study focuses on the water supply system in northern Taiwan, covering Hsinchu, Taoyuan, New Taipei, and Taipei. Water is temporarily stored in reservoirs and diversion weirs along seven major rivers, including the Beishi, Tonghou, Nanshi, Sanxia, Dahan, Touqian, and Shangping Rivers, and is then treated and conveyed to the target cities, as shown in Figure 1.

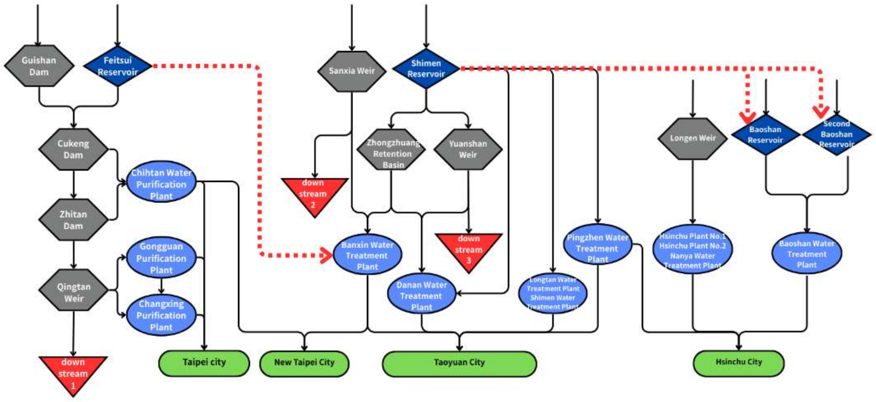

Figure 1: Northern Taiwan Water Supply Network System

Interconnecting trunk pipelines across regions presents considerable challenges in evaluating capacity and system stability. To enhance supply reliability and scheduling efficiency, this study focuses on optimizing Taiwan’s existing and planned water dispatch networks while proposing scientific strategies for cross-regional integration. These strategies aim to improve system stability, support drought mitigation through pipeline development and integrated use of artificial lakes and groundwater, and ensure the sustainable development of agriculture and the high-tech sector. Beyond theoretical modeling, this research provides data-driven support for policy formulation and infrastructure planning, offering practical insights to address Taiwan’s water resource challenges under climate change.

二、研究方法

This study analyzes two scenarios based on the hydrological conditions of 2020, a year marked by severe drought. The first scenario models the water system using only the infrastructure that was already in place. The second incorporates both existing facilities and the planned interconnecting pipelines. The corresponding mathematical formulations are presented below.

This study develops a minimal cost network programming model integrated with spectrum analysis to establish a comprehensive framework for interconnecting water supply systems. The minimal cost network programming model retains fundamental concepts from traditional network models, such as flow conservation and resource allocation, while also incorporating several practical constraints encountered in real-world transport processes, including path capacity limits and marginal cost functions related to flow. Spectrum analysis further transforms the problem into the frequency domain, where unstable supply-demand conditions are decomposed into distinct frequencies representing different levels of water intensity. The mathematical formulation in the frequency domain is as follows:

$$\min\{z\} = \left(a_0^{out} - a_0^{Demand}\right)^2 + \sum_{n=1}^{N}\left[\left(a_n^{out} - a_n^{Demand}\right)^2 + \left(b_n^{out} - b_n^{Demand}\right)^2\right] \tag{1}$$

$$\text{S.T.} \quad \frac{T}{2\pi n} b_n^{s_i} = \sum_j a^{f_{ji}} - \sum_k a^{f_{ik}} \qquad \forall\, n \in 1, \dots, N;\ \forall i \in N;\ \forall\, (ji) \cup (ik) \in E \tag{2}$$

$$\frac{T}{2\pi n} a_n^{s_i} = \sum_j b^{f_{ji}} - \sum_j b^{f_{ik}} \qquad \forall\, n \in 1, \dots, N;\ \forall i \in N;\ \forall\, (ji) \cup (ik) \in E \tag{3}$$

$$L_{ij} \le a_0^{f_{ij}} + \sum_{n=1}^{N}\left[a_n^{f_{ij}} \cos\left(\frac{2\pi}{T} nt\right) + b_n^{f_{ij}} \sin\left(\frac{2\pi}{T} nt\right)\right] \le U_{ij} \qquad \forall\, t = [0, T] \tag{4}$$

$$0 \le a_0^{s_i} + \sum_{n=1}^{N}\left[a_n^{s_i} \cos\left(\frac{2\pi}{T} nt\right) + b_n^{s_i} \sin\left(\frac{2\pi}{T} nt\right)\right] \le S_i \qquad \forall\, t = [0, T] \tag{5}$$

$$\text{Where} \quad a^{out} = \sum_i \sum_j a^{f_{ji}} \qquad \forall\, i \in N \text{ such that } D_i \neq 0; \forall\, (ji) \in E \tag{6}$$

$$b^{out} = \sum_i \sum_j b^{f_{ji}} \qquad \forall\, i \in N \text{ such that } D_i \neq 0; \forall\, (ji) \in E \tag{7}$$

$$a^{Demand} = a^{D_i} \qquad \forall\, i \in N \text{ such that } D_i \neq 0 \tag{8}$$

$$b^{Demand} = b^{D_i} \qquad \forall\, i \in N \text{ such that } D_i \neq 0 \tag{9}$$

In the network flow model, the system is represented as a directed graph $G(N, E)$, where N denotes the set of nodes. Any three nodes $(j, i, k) \in N$ can form directed edges $(ji) \cup (ik) \in E$, indicating inflow from node j to i and outflow from i to k. Equation (1) is the objective function that minimizes the total squared difference between water supply and demand across all regions. Here, f_{ji} represents the flow from node j to node i, and D_i denotes the demand at node i. According to Plancherel and Leffler (1910), Plancherel's theorem states that the energy of a signal is preserved between the time and frequency domains. Therefore, the squared difference in supply and demand can be decomposed via Fourier expansion into N frequency components, each calculated independently over orthogonal sine and cosine bases.

From Equation (2) onward, the model constraints are introduced. Equations (2) and (3) represent the flow balance conditions at each node i, where the net storage s_i equals the sum of inflows minus outflows, expressed in sine and cosine terms due to their linear independence. Equation (4) defines the flow boundary condition on each edge $(i, j) \in E$, where the flow f_{ij} must lie between a minimum L_{ij} and a maximum U_{ij}, with f_{ij} expanded using Fourier series. Equation (5) sets the boundary condition for node storage, where storage s_i must be non-negative and less than its maximum capacity s_i, and is also represented in Fourier form.

三、結果與討論

This study presents the optimized water supply and demand conditions for four counties under two different system configurations during the 2020 extreme drought. The demand, comprising domestic, industrial, and agricultural uses, is assessed under Scenario 1 with existing infrastructure and Scenario 2 with both current and planned interconnecting pipelines.

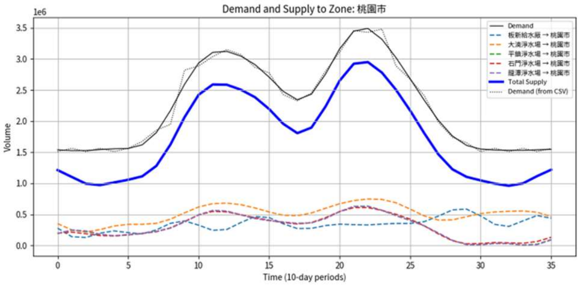

Figure 2: Water supply and demand in Taoyuan City under Scenario 1

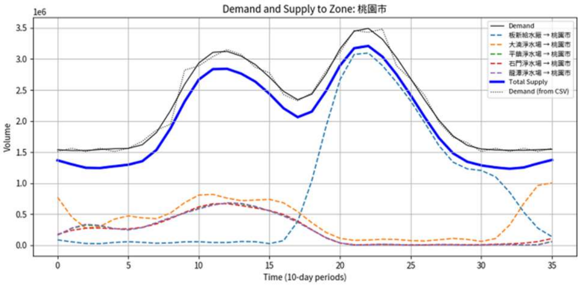

Figure 3: Water supply and demand in Taoyuan City under Scenario 2

As shown in Figure *2* and Figure 3, both system configurations result in insufficient supply during the extreme drought. However, the proposed model distributes the water deficit evenly across time, avoiding sharp shortages in specific periods. This approach reduces the risk of acute water crises by adopting a strategy of prolonged but moderate shortages. Additionally, Scenario 2 shows a smaller supply-demand gap compared to Scenario 1, indicating that if the planned infrastructure is completed, the proposed strategy could significantly enhance water supply stability in future drought events.

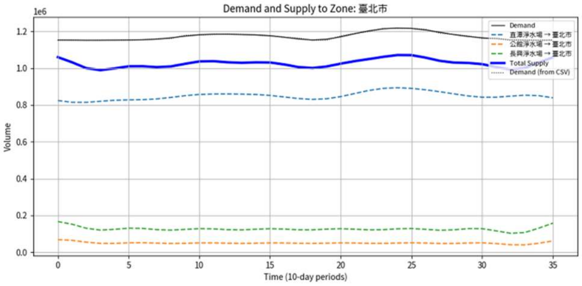

Figure 4: Water supply and demand in Taipei City under Scenario 1

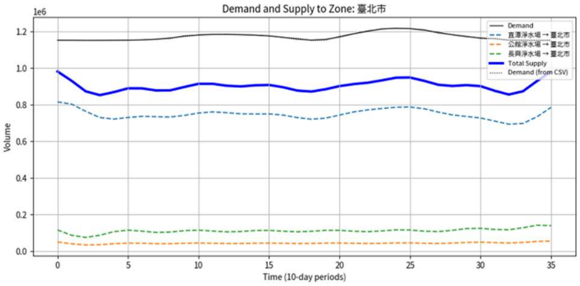

Figure 5: Water supply and demand in Taipei City under Scenario 2

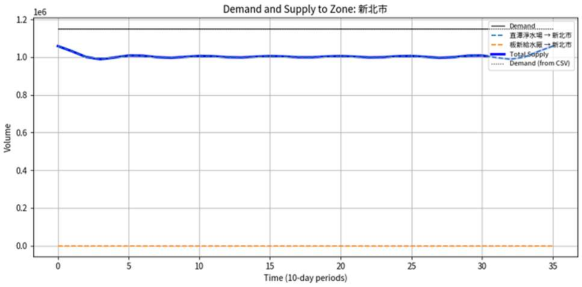

Figure 6: Water supply and demand in New Taipei City under Scenario 1

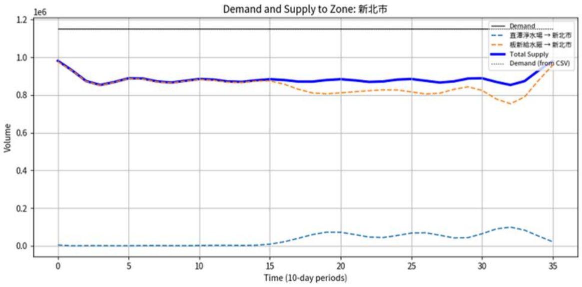

Figure 7: Water supply and demand in New Taipei City under Scenario 2

Figure 4 to Figure 7 illustrate the supply-demand conditions in Taipei and New Taipei during the 2020 extreme drought under two network configurations. Both cities face persistent shortages even with optimized strategies. In Scenario 2, including the planned interconnecting pipelines, deficits in both cities increase by around 150,000 cubic meters per ten-day period. This reallocation improves Taoyuan's supply by approximately 300,000 cubic meters per period, as its shortfall exceeds the combined deficits of the other two cities.

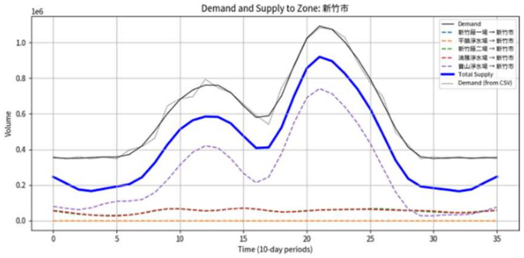

Figure 8: Water supply and demand in Hsinchu City under Scenario 1

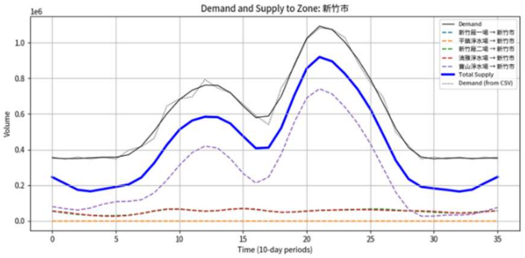

Figure 9: Water supply and demand in Hsinchu City under Scenario 2

Figure 8 and Figure 9 present Hsinchu’s supply-demand conditions under the same drought scenario and network configurations. Although the planned pipelines were designed to supply Hsinchu from the Taoyuan system, no water is delivered due to Taoyuan’s severe shortage. As a result, the allocation strategy in Hsinchu remains unchanged.

四、結論

This study evaluated northern Taiwan’s water allocation during the 2020 extreme drought under two system configurations: existing infrastructure and planned interconnecting pipelines. While shortages persist, pipelines improve supply in severely affected areas by balancing regional gaps, though transfers may raise deficits elsewhere, highlighting the need for clear allocation priorities. Using a network flow model with Fourier spectrum analysis, the study quantifies supply-demand functions, assesses cross-regional benefits, and provides a framework for multi-scenario planning and climate resilience.

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